

NEW CONCEPTS OF INSTANTANEOUS ACTIVE AND REACTIVE POWERS IN ELECTRICAL SYSTEMS WITH GENERIC LOADS

Edson H. Watanabe, Member, IEEE Richard M. Stephan, Member, IEEE and Maurício Aredes

COPPE/Federal University of Rio de Janeiro
Caixa Postal 68504
21945 - Rio de Janeiro, RJ, Brazil

Keywords: Instantaneous Active and Reactive Power;
Harmonic Power; Active Power Filters.

ABSTRACT

This paper presents first a review of the conventional active and reactive power theory, valid for the steady-state analysis and then the instantaneous power theory, introduced by Akagi et al [1, 2], is presented. This instantaneous theory is, valid for steady-and-transient states and for generic voltage and current waveforms. Some examples explaining the physical meaning of the new concepts will be presented. Also, by using the well known concepts of symmetrical components together with the new theory the powers in an unbalanced system will be analyzed, including the zero-sequence instantaneous power. It will be presented an example showing how this theory can be used to design and control an active power filter. Some simulation results are presented and discussed.

INTRODUCTION

The active and reactive power for electric circuit with sinusoidal source and linear loads are well established, (e.g. Elgerd [3]). In the cases of non-linear loads these concepts are not yet sufficiently explained and with the increased use of power electronics circuits, the electric sources had to start supplying power to a large number of these loads. Therefore, nowadays, understanding the reactive and harmonic power is an actual necessity for understanding and accomplishing reactive compensation or harmonic filtering. In fact, this problem has been treated in various previous work [4]-[7]. However, the authors believe that only after the work presented by Akagi et al. in [1] and [2] a new and concise theory to deal with this problem actually appeared. This theory is well known in Japan as "instantaneous power theory" or "p-q theory" and has been used to develop many types of active power filters (e.g. [2] and [8]). More recently, Furuhashi et al. [9] presented, also, a study on the theory of instantaneous reactive power, but it did not consider the "active alternating power" and the zero-sequence power accurately.

In the present paper, first the conventional active and reactive power theory, which is valid for steady-state analysis will be reviewed, and then the new concepts of instantaneous real and imaginary power, as introduced by Akagi et al. in [1, 2], valid for steady-and-transient states and for generic voltage and current waveforms, will be presented. Some simple example will be presented to better explain the physical meaning of the real and imaginary powers, including their alternating parts [1, 2]. Also, by using the well known

92 WM 279-0 PWRD A paper recommended and approved by the IEEE Transmission and Distribution Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1992 Winter Meeting, New York, New York, January 26 - 30, 1992. Manuscript submitted August 29, 1991; made available for printing November 27, 1991.

concepts of symmetrical components the powers in an unbalanced system will be analyzed, and the physical meaning of the zero-sequence instantaneous power will be presented. It will be also presented an example showing how this theory can be used to design and control an active power filter for a balanced system with harmonics. Some simulation results are presented and discussed.

CONVENTIONAL CONCEPTS

For a better understanding of the new concepts, first the conventional concepts, which is valid for the steady state, will be presented.

Sinusoidal Voltage Source and Linear Loads

- a. Single-phase case - For this study it is assume that the voltage source and the load current are given by

$$v_a(t) = \sqrt{2} V \sin \omega t \quad \text{and} \quad i_a(t) = \sqrt{2} I \sin(\omega t - \phi). \quad (1)$$

The instantaneous power can be calculated by

$$p_a(t) = v_a i_a = \underbrace{VI \cos \phi (1 - \cos 2\omega t)}_{\text{"1"}} - \underbrace{VI \sin \phi \sin 2\omega t}_{\text{"2"}} \quad (2)$$

This decomposition shows that the instantaneous power can be separated in two parts:

part "1" has an average value equal to $VI \cos \phi$ and has an alternating component on it, oscillating at twice the line frequency. This part "1" never becomes negative and, therefore, is an unidirectional (dc) power [4], [7].

part "2" is also an alternating component, oscillating at twice the line frequency, has peak value equal to $VI \sin \phi$ and zero average value.

The average (active) power is, therefore, given by

$$P = VI \cos \phi \quad (3)$$

and the conventional reactive power is just defined as the peak value of part "2" (power component which average value is zero), or

$$Q \triangleq VI \sin \phi. \quad (4)$$

Now we can re-write (2) as

$$p_a(t) = P (1 - \cos 2\omega t) - Q \sin 2\omega t. \quad (5)$$

- b. Three-phase case - Considering a balanced, three-phase system with phases a, b and c and linear loads, we may conclude the following:

- (i) The instantaneous three-phase active power is given by

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t) = 3P \triangleq P_{3\phi}. \quad (6)$$

That is, the instantaneous three-phase power is constant and, therefore equal to its average value.

- (ii) The alternating components of the instantaneous power, which are dependent of Q (part "2" in (2)), are phase shifted by 120° from phase to phase and therefore their sum is zero. However, using the conventional concepts, the reactive power in a three-phase system is defined as

$$Q_{3\phi} \triangleq 3Q \tag{7}$$

just because this type of power exists in each phase independently. It is important to note that this reactive power can only be calculated by definition, since its instantaneous sum in the three-phases is equal to zero.

Sinusoidal Voltage Source and Non-linear Loads

In this case the voltage source is the same as given in (1), but the current contains harmonics at frequencies multiples of ω , that is

$$i_a(t) = \sum_{n=1}^{\infty} \sqrt{2} I_n \sin(n\omega t - \phi_n). \tag{8}$$

The following relations are known:

- Instantaneous Power:

$$p_a(t) = VI_1 \cos \phi_1 (1 - \cos 2\omega t) - VI_1 \sin \phi_1 \sin 2\omega t + \sum_{n=2}^{\infty} 2VI_n \sin \omega t \sin(n\omega t - \phi_n). \tag{9}$$

- Average active power:

$$P = \text{average value of } p_a(t) = VI_1 \cos \phi_1 \tag{10}$$

- Current rms value:

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots} = \sqrt{(1/T) \int_0^T i_a^2 dt} \tag{11}$$

where T is the period of $i_a(t)$.

- Apparent Power:

$$S = VI \tag{12}$$

From (11) and (12) it follows that:

$$S^2 = V^2 I^2 = V^2 (I_1^2 + I_2^2 + I_3^2 + \dots). \tag{13}$$

In this system, with sinusoidal voltage source and non-linear load (current harmonics at frequencies multiples of ω), the reactive power is conventionally defined as

$$Q \triangleq VI_1 \sin \phi_1 \tag{14}$$

and the harmonic power as

$$H \triangleq V \sqrt{I_2^2 + I_3^2 + \dots} \tag{15}$$

Now, (13) can be re-written as

$$S^2 = P^2 + Q^2 + H^2. \tag{16}$$

This result is normally represented using the power tetrahedron, instead of the triangle as in the linear case, as shown in Fig. 1 [5], [10]. From this Fig. 1 various important factors can be determined:

- Displacement factor or fundamental power factor = $\cos \phi_1$,

- Distortion factor = $\sqrt{P^2 + Q^2} / S = I_1 / I = \cos \gamma$,

- Power factor or total power factor = $P / S = \cos \phi_1 \cos \gamma = \cos \phi$.

The displacement factor corresponds to the power factor of systems without harmonics. This factor may be called fundamental power factor, as it depends only on the current fundamental component. On the other hand, the power factor, as defined above, may be called total power factor, as it depends on fundamental and all harmonic components.

For a three-phase balanced system all the quantities in (10), (12), (14) and (15) should be multiplied by 3 and the following can be pointed out:

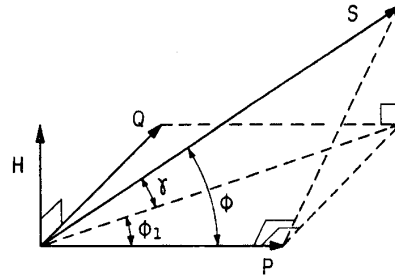


Fig. 1 - Power Tetrahedron

- i) P and Q are dependent only on the current component at frequency ω ;
- ii) H is dependent on the current components with frequencies different from ω (harmonics);
- iii) The power component given by $VI_1 \sin \phi_1 \sin 2\omega t$ or $Q \sin 2\omega t$ has zero average value and can be eliminated by using a conveniently chosen capacitor or inductor. The connection of an L or C component in parallel with the load allows the generation of a current at frequency ω that absorbs or generates the reactive power Q required by the load. Note that with this procedure it is not possible to generate or to absorb the harmonic power, H, since it depends on frequencies different from ω (harmonics).
- iv) The power components of (9) that depends on frequencies different from ω , that is the parts that corresponds to H, have average value equal to zero and can not be eliminated by a single capacitor or inductor. The elimination of H depends on filters that work as a short-circuit for the harmonic current generated by the load.

NEW CONCEPTS OF ACTIVE AND REACTIVE POWER

In the previous section, the power components of an electric circuit have been arranged in a tetrahedron. The reactive and harmonic power compensation have been discussed, but the theory considered basically the steady-state behaviour. As the load may change continuously and the harmonic contents, too, a theory for transient conditions is necessary. Akagi et al. [1, 2] have proposed new concepts of instantaneous active and reactive power, that can be used in transient states and when the voltage and/or current have generic waveforms. This theory has some interesting but difficult aspects that need to be clarified. In the following, this theory, valid for three-phase 4-wires system, will be summarized and the meaning of each term explained.

The Generalised Theory of Instantaneous Power [1, 2]

The α - β -0 transformation of a three-phase four wires system gives:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \tag{17}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix}, \tag{18}$$

where v_a, v_b, v_c are phase voltages. Identical relations hold for line currents i_a, i_b and i_c .

The instantaneous three-phase active power is given by:

$$\begin{aligned} p_{3\phi}(t) &= v_a i_a + v_b i_b + v_c i_c = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \\ &= p_a(t) + p_b(t) + p_c(t) = p_\alpha(t) + p_\beta(t) + p_0(t) \\ &= p(t) + p_0(t), \end{aligned} \quad (19)$$

where:

$p = p_\alpha + p_\beta$ is the instantaneous real power; and

$p_0 = v_0 i_0$ is the instantaneous zero-sequence power. The physical meaning of these two powers will be explained later.

One advantage of using the α - β -0 transformation is to separate the zero-sequence component of the system.

Akagi et al. [1, 2] suggest the definition of the following variable:

$$q(t) \triangleq v_\alpha i_\beta - v_\beta i_\alpha. \quad (20)$$

In fact this variable can be re-written in terms of a-b-c components:

$$q = -[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b]/\sqrt{3}. \quad (21)$$

This is a well known expression used for measurement of the conventional three-phase reactive power, when only the fundamental frequency is considered. The variable q , in the new concept, considers all the frequency components in currents and voltages. Therefore, it has, now, a different physical meaning. This is the reason that made Akagi et al. call it as "instantaneous imaginary power", defining also a new unit IVA (Imaginary Volt-Ampere). From (20) it can be seen that q is not influenced by zero-sequence components of the system, since it depends only on α and β components.

The powers p and q can be re-written as

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \quad (22)$$

From this matricial equation, for $\Delta = v_\alpha^2 + v_\beta^2$, it follows that:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}. \quad (23)$$

Or, separating the parts as functions of p and q :

$$\begin{aligned} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} &= \frac{1}{\Delta} \left\{ \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} + \right. \\ &\left. + \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \right\} \triangleq \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix}, \end{aligned} \quad (24)$$

where, the current components are:

$$i_{\alpha p} = v_\alpha p / \Delta, \quad i_{\alpha q} = -v_\beta q / \Delta, \quad (25)$$

$$i_{\beta p} = v_\beta p / \Delta, \quad i_{\beta q} = v_\alpha q / \Delta. \quad (26)$$

Then, the power in phases α and β can be separated as:

$$\begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix} = \begin{bmatrix} v_\alpha i_\alpha \\ v_\beta i_\beta \end{bmatrix} = \begin{bmatrix} v_\alpha i_{\alpha p} \\ v_\beta i_{\beta p} \end{bmatrix} + \begin{bmatrix} v_\alpha i_{\alpha q} \\ v_\beta i_{\beta q} \end{bmatrix} \triangleq \begin{bmatrix} p_{\alpha p} \\ p_{\beta p} \end{bmatrix} + \begin{bmatrix} p_{\alpha q} \\ p_{\beta q} \end{bmatrix}, \quad (27)$$

where the power components are:

$$p_{\alpha p} = v_\alpha i_{\alpha p} = v_\alpha^2 p / \Delta, \quad (28)$$

$$p_{\alpha q} = v_\alpha i_{\alpha q} = -v_\alpha v_\beta q / \Delta, \quad (29)$$

$$p_{\beta p} = v_\beta i_{\beta p} = v_\beta^2 p / \Delta, \quad (30)$$

$$p_{\beta q} = v_\beta i_{\beta q} = v_\alpha v_\beta q / \Delta. \quad (31)$$

Therefore the three-phase active power can be re-written:

$$\begin{aligned} p_{3\phi} &= p_\alpha + p_\beta + p_0 = p_{\alpha p} + p_{\alpha q} + p_{\beta p} + p_{\beta q} + p_0 = \\ &= p_{\alpha p} + p_{\beta p} + p_0, \end{aligned} \quad (32)$$

since from (29) and (31):

$$0 = p_{\alpha q} + p_{\beta q}. \quad (33)$$

These relations suggest the nomenclature below:

$$\begin{aligned} p_{\alpha p} &= \alpha\text{-axis instantaneous active power,} \\ p_{\beta p} &= \beta\text{-axis instantaneous active power,} \\ p_{\alpha q} &= \alpha\text{-axis instantaneous reactive power,} \\ p_{\beta q} &= \beta\text{-axis instantaneous reactive power.} \end{aligned}$$

It is important to observe that these quantities are all instantaneous and valid for transient or steady state, and harmonics may be present in voltage and/or current. In the conventional concepts the reactive power corresponds to the peak value of the parcel of the instantaneous power whose mean-value is zero (part "2" in (2)). Now, in the new concepts the reactive power corresponds to the parts of the instantaneous power, that is dependent on the instantaneous imaginary power q , exists in each phase independently, but vanishes when added ($p_{\alpha q} + p_{\beta q} = 0$), in a two-phase (α - β) system. If these powers $p_{\alpha q}$ and $p_{\beta q}$ are transformed back to the original three-phase (a - b - c) system, they also vanish when added.

The instantaneous real power p , gives the net energy per second being transported from source to load and vice-versa at any time. This real power is, by definition, dependent only on the voltage and currents in phases α and β . There is no zero-sequence power in it.

Three-phase Sinusoidal Voltage Supplying a Linear Load

To establish a connection with the well known conventional active (P) and reactive (Q) power, in this Section a balanced linear load supplied by a three-phase balanced sinusoidal voltage will be analysed. Let the voltages and currents be:

$$\begin{aligned} v_a &= \sqrt{2} V \sin \omega t, & i_a &= \sqrt{2} I \sin (\omega t - \phi), \\ v_b &= \sqrt{2} V \sin (\omega t + 120^\circ), & i_b &= \sqrt{2} I \sin (\omega t + 120^\circ - \phi), \\ v_c &= \sqrt{2} V \sin (\omega t - 120^\circ), & i_c &= \sqrt{2} I \sin (\omega t - 120^\circ - \phi). \end{aligned} \quad (34)$$

Then:

$$\begin{aligned} v_\alpha &= \sqrt{3} V \sin \omega t, & i_\alpha &= \sqrt{3} I \sin (\omega t - \phi), \\ v_\beta &= \sqrt{3} V \cos \omega t, & i_\beta &= \sqrt{3} I \cos (\omega t - \phi), \\ v_0 &= 0, & i_0 &= 0. \end{aligned} \quad (35)$$

Therefore, from (22)

$$p = v_\alpha i_\alpha + v_\beta i_\beta = 3 VI \cos \phi = P_{3\phi} \quad (36)$$

$$q = v_\alpha i_\beta - v_\beta i_\alpha = 3 VI \sin \phi = Q_{3\phi}. \quad (37)$$

These equations show the equivalence of the traditional concepts of active and reactive power and the new ones.

Next, a three-phase balanced source supplying a non-linear load will be studied.

Three-phase Sinusoidal Voltage Supplying a Non-linear Load

Suppose an electrical system with the voltages sources as given in (34) and with the following currents:

$$\begin{aligned}
 i_a &= \sum_{n=1}^{\infty} \sqrt{2} I_n \sin(n\omega t - \phi_n), \\
 i_b &= \sum_{n=1}^{\infty} \sqrt{2} I_n \sin[n(\omega t + 120^\circ) - \phi_n], \\
 i_c &= \sum_{n=1}^{\infty} \sqrt{2} I_n \sin[n(\omega t - 120^\circ) - \phi_n].
 \end{aligned} \quad (38)$$

Then,

$$\begin{aligned}
 i_\alpha &= \sum_{n=1}^{\infty} \frac{2}{\sqrt{3}} I_n \sin(n\omega t - \phi_n) [1 - \cos(n120^\circ)], \\
 i_\beta &= \sum_{n=1}^{\infty} 2 I_n \cos(n\omega t - \phi_n) \sin(n120^\circ), \\
 i_0 &= \frac{1}{\sqrt{3}} (i_a + i_b + i_c) = \sum_{n=1}^{\infty} \sqrt{6} I_{3n} \sin(3n\omega t - \phi_{3n}).
 \end{aligned} \quad (39) \quad (40) \quad (41)$$

It is interesting to note that the harmonics of order "3n" appear in i_0 .

The power components p , q , p_0 and $p_{3\phi}$ are:

$$\begin{aligned}
 p &= v_\alpha^i i_\alpha^j + v_\beta^j i_\beta^i = p_{\alpha p} + p_{\beta p} = 3V I_1 \cos \phi_1 - \\
 &\quad - 3V I_2 \cos(3\omega t - \phi_2) + 3V I_4 \cos(3\omega t + \phi_4) - \\
 &\quad - 3V I_5 \cos(6\omega t - \phi_5) + 3V I_7 \cos(6\omega t + \phi_7) - \dots
 \end{aligned} \quad (42)$$

$$\begin{aligned}
 q &= v_\alpha^j i_\beta^i - v_\beta^i i_\alpha^j = 3V I_1 \sin \phi_1 - \\
 &\quad - 3V I_2 \sin(3\omega t - \phi_2) + 3V I_4 \sin(3\omega t + \phi_4) - \\
 &\quad - 3V I_5 \sin(6\omega t - \phi_5) + 3V I_7 \sin(6\omega t + \phi_7) - \dots
 \end{aligned} \quad (43)$$

$$p_0 = v_0 i_0 = 0 \quad \text{and} \quad p_{3\phi} = p. \quad (44)$$

Observing these equations, it is reasonable to write:

$$p = \bar{p} + \tilde{p} \quad \text{and} \quad q = \bar{q} + \tilde{q}, \quad (45)$$

where "-" indicates "mean-value" and "~" alternating components with mean-value equal to zero.

From (42) and (43) it is possible to conclude that:

$$\bar{p} = P_{3\phi} \quad \text{and} \quad \bar{q} = Q_{3\phi}, \quad (46)$$

$$H = \sqrt{\bar{P}^2 + \bar{Q}^2}, \quad (47)$$

where \bar{P} and \bar{Q} are the root mean square values of \tilde{p} and \tilde{q} , respectively.

The above equations give the relationship between the new and the conventional power theory. The average value of p , that is \bar{p} , in this example, corresponds to the conventional average power. The real alternating power \tilde{p} represents the energy per second that is being transported from source to load or vice-versa at any time, due to current harmonics and its average value is zero. This pulsation of energy between source and load represents the energy being stored or released at the three-phase or two-phase (α - β) load or source. The average value of the imaginary power q , that is \bar{q} , corresponds to the conventional reactive power. The alternating part of q , i.e. \tilde{q} , is responsible for the harmonic (due to the current) reactive power in each phase, but vanishes instantaneously when added.

The imaginary power $q = \bar{q} + \tilde{q}$ does not contribute for the instantaneous energy transport, although the reactive currents (see (25) and (26)) exists in each phase and obviously occupies part of the conductor area.

From (47) it is possible to see that the conventional harmonic power H is composed by the alternating real and imaginary powers.

Nomenclature Expansion

Based on (25) and (26) and considering the definitions given in (45), the following new components are introduced: $i_{\alpha p}^{\sim}$, $i_{\alpha q}^{\sim}$, $i_{\beta p}^{\sim}$, $i_{\beta q}^{\sim}$, $i_{\alpha p}^{\bar{}}$, $i_{\alpha q}^{\bar{}}$, $i_{\beta p}^{\bar{}}$, $i_{\beta q}^{\bar{}}$. For instance,

$i_{\alpha p}^{\sim} = v_\alpha^{\sim} \tilde{p} / \Delta$. The other terms can be obtained similarly.

Based on (28) to (31) the power components can also be expanded to: $p_{\alpha p}^{\sim}$, $p_{\alpha q}^{\sim}$, $p_{\beta p}^{\sim}$, $p_{\beta q}^{\sim}$, $p_{\alpha p}^{\bar{}}$, $p_{\alpha q}^{\bar{}}$, $p_{\beta p}^{\bar{}}$, $p_{\beta q}^{\bar{}}$.

Unbalanced Three-Phase 4-Wire System

To better understand how important are the new concepts an example considering an unbalanced three-phase 4-wire voltage source supplying an linear load will be analyzed. For this analysis it will be assumed that the system is in steady-state condition and that \dot{V}_a , \dot{V}_b , \dot{V}_c and \dot{I}_a , \dot{I}_b , \dot{I}_c are respectively the voltage and current phasors for each phase. The symmetrical components of these voltages are given by:

$$\begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix} \quad (48)$$

where, "0", "+" and "-" subscripts denotes zero, positive and negative sequences, respectively. The α operator is equal to $e^{j2\pi/3}$. Similar relations hold for the load currents.

The inverse symmetrical components transformation of (48) is:

$$\begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} \quad (49)$$

Now, it is possible to write the time functions corresponding to the phasors defined in (49). That is,

$$\begin{aligned}
 v_a &= \sqrt{2} V_0 \sin(\omega t + \phi_0) + \sqrt{2} V_+ \sin(\omega t + \phi_+) + \sqrt{2} V_- \sin(\omega t + \phi_-), \\
 v_b &= \sqrt{2} V_0 \sin(\omega t + \phi_0) + \sqrt{2} V_+ \sin(\omega t - 120^\circ + \phi_+) + \\
 &\quad + \sqrt{2} V_- \sin(\omega t + 120^\circ + \phi_-); \\
 v_c &= \sqrt{2} V_0 \sin(\omega t + \phi_0) + \sqrt{2} V_+ \sin(\omega t + 120^\circ + \phi_+) + \\
 &\quad + \sqrt{2} V_- \sin(\omega t - 120^\circ + \phi_-).
 \end{aligned} \quad (50)$$

The transformation of the instantaneous voltages in (50) in α - β -0 axis as in (17) gives:

$$\begin{aligned}
 v_\alpha &= \sqrt{3} V_+ \sin(\omega t + \phi_+) + \sqrt{3} V_- \sin(\omega t + \phi_-), \\
 v_\beta &= -\sqrt{3} V_+ \cos(\omega t + \phi_+) + \sqrt{3} V_- \cos(\omega t + \phi_-), \\
 v_0 &= \sqrt{6} V_0 \sin(\omega t + \phi_0).
 \end{aligned} \quad (51)$$

Similarly, for the load currents \dot{I}_a , \dot{I}_b and \dot{I}_c it is possible to obtain their instantaneous α - β -0 transformation, i.e.,

$$\begin{aligned}
 i_\alpha &= \sqrt{3} I_+ \sin(\omega t + \delta_+) + \sqrt{3} I_- \sin(\omega t + \delta_-) \\
 i_\beta &= -\sqrt{3} I_+ \cos(\omega t + \delta_+) + \sqrt{3} I_- \cos(\omega t + \delta_-) \\
 i_0 &= \sqrt{6} I_0 \sin(\omega t + \delta_0)
 \end{aligned} \quad (52)$$

In (51) and (52) it is possible to see that positive and negative sequences appear only in the α and β axis. On the other hand, zero-sequence components appear only in the "0" axis. Using these voltages and currents it is possible to calculate the expressions for the real (p), imaginary (q) and zero-sequence (p_0) instantaneous powers. These powers can be separated in their average values and alternating parts. That is,

$$\begin{aligned}
 \bar{p} &= 3V_+ I_+ \cos(\phi_+ - \delta_+) + 3V_- I_- \cos(\phi_- - \delta_-), \\
 \tilde{p} &= 3V_+ \tilde{I}_+ \cos(\phi_+ - \delta_+) + 3V_- \tilde{I}_- \cos(\phi_- - \delta_-),
 \end{aligned} \quad (53)$$

– Instantaneous average imaginary power:

$$\bar{q} = -3V_+ I_+ \sin(\phi_+ - \delta_+) + 3V_- I_- \sin(\phi_- - \delta_-), \quad (54)$$

– Instantaneous alternating real power:

$$\tilde{p} = -3V_+ I_+ \cos(2\omega t + \phi_+ + \delta_-) - 3V_- I_- \cos(2\omega t + \phi_- + \delta_+), \quad (55)$$

– Instantaneous alternating imaginary power:

$$\tilde{q} = 3V_+ I_+ \sin(2\omega t + \phi_+ + \delta_-) - 3V_- I_- \sin(2\omega t + \phi_- + \delta_+), \quad (56)$$

– Instantaneous average zero-sequence power:

$$\bar{p}_0 = 3V_0 I_0 \cos(\phi_0 - \delta_0), \quad (57)$$

– Instantaneous alternating zero-sequence power:

$$\tilde{p}_0 = -3V_0 I_0 \cos(2\omega t + \phi_0 + \delta_0). \quad (58)$$

From the above expressions, some important conclusions must be pointed out:

- (i) The zero-sequence power p_0 is exactly equal to the power in a single-phase circuit as shown in (2), having an average part and an alternating component. Its characteristics is similar to the real power p , that is, the value of the zero-sequence power at any time gives the amount of energy being transported per second from source to load and vice-versa. The alternating component of p_0 has a relatively low frequency (2ω). In terms of the new concepts, there is no reactive power in p_0 .
- (ii) Both positive and negative sequence produces \bar{p} and \bar{q} , however their simultaneous presence produces alternating components \tilde{p} and \tilde{q} , even though no harmonic component was considered in voltage or current.
- (iii) Only positive and negative sequences can produce reactive power, in terms of the new concepts, that is the power that exists in each phase but their instantaneous sum is zero.

The above study can be generalized for the cases where harmonic components are present in the phase voltages and currents. However it will not be introduced here due to space limitation.

REACTIVE AND HARMONIC POWER COMPENSATION

In this section it will be presented an example of reactive and harmonic power compensation. Initially, the compensation of the power terms that depend on q (i.e., $p_{\alpha q}$ and $p_{\beta q}$) will be considered. Fig. 2 illustrates the power flow in phases α and β . To compensate completely $p_{\alpha q}$ and $p_{\beta q}$, it is necessary to introduce the current sources $i_{\alpha c}$ and $i_{\beta c}$, shown in Fig. 2, such that:

$$i_{\alpha c} = i_{\alpha q}, \quad \text{and} \quad (59)$$

$$i_{\beta c} = i_{\beta q}, \quad (60)$$

where $i_{\alpha q}$ and $i_{\beta q}$ are given by (25) and (26). The current source $i_{\alpha c}$ is in parallel with the voltage source v_α and, therefore, supplies the power $p_{\alpha q} = v_\alpha i_{\alpha q}$ (see (29)). Similarly the current source $i_{\beta c}$ supplies $p_{\beta q} = v_\beta i_{\beta q}$ (see (31)). Then, the voltage sources v_α and v_β need to supply only $p_{\alpha p}$ and $p_{\beta p}$. In all time instants $p_{\alpha q} + p_{\beta q} = 0$, which means that the power necessary to compensate for $i_{\alpha q}$ is equal to the negative of the power necessary to compensate for $i_{\beta q}$.

The current sources $i_{\alpha c}$ and $i_{\beta c}$ represent active power filters, that may be voltage source inverters conveniently controlled to generate $i_{\alpha q}$ and $i_{\beta q}$ [2]. If a common DC source is used to supply these inverters, or one two-phase inverter is used, it is possible to conclude that instantaneously there is no

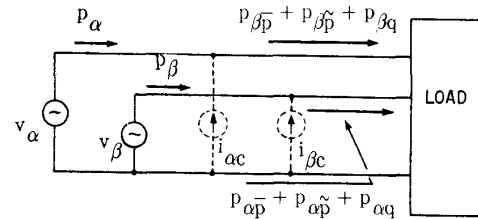


Fig. 2 – Compensation scheme.

power flowing out of or into this DC source. Therefore, no DC source is necessary and also no energy storage element is necessary to compensate for these power terms [2]. Instantaneously, the reactive power required by one phase can be supplied by the other one. In fact, in actual systems only a small capacitor is used because the switching of the inverters [2]. It is important to note that this example is for the compensation of q , that means compensation of the conventional reactive power Q ($=\bar{q}$) and also the harmonic imaginary power \tilde{q} .

Now, the power terms that depend on p (i.e. $p_{\alpha p}$, $p_{\beta p}$) will be analysed. As shown in (45), p can be decomposed in two parts: \bar{p} and \tilde{p} . Since \bar{p} represents the power that is effectively converted into work and therefore has to be supplied by the source, only \tilde{p} may be compensated.

According to (28) and (30), it is possible to write:

$$p_{\alpha \tilde{p}} = v_\alpha^2 \tilde{p} / \Delta = v_\alpha i_{\alpha \tilde{p}}, \quad (61)$$

$$p_{\beta \tilde{p}} = v_\beta^2 \tilde{p} / \Delta = v_\beta i_{\beta \tilde{p}}. \quad (62)$$

These terms have mean-value equal to zero, but their sum is not zero at all instants, i.e. $p_{\alpha \tilde{p}} + p_{\beta \tilde{p}} \neq 0$. Therefore, to compensate for $p_{\alpha \tilde{p}}$, there is no need of power supply for the inverters, however energy storage elements are necessary. In this case, it is not possible to supply the compensating power from one phase to the other one, as was viable with $p_{\alpha q}$ and $p_{\beta q}$. The energy storage element receives energy when \tilde{p} is negative and supplies when \tilde{p} is positive. This compensation can be implemented with active power filters based on inverters conveniently controlled to supply $i_{\alpha \tilde{p}}$ and $i_{\beta \tilde{p}}$, as given by (61) and (62) [8]. The basic difference with the compensation of q is that now an energy storing element, capacitor or inductor has to be employed to store this active energy oscillation.

Of course, if all the harmonics has to be filtered out \tilde{p} and \tilde{q} has to be eliminated.

Comparison With the Conventional Compensation

The compensation method presented above differs significantly from the traditional one. Usually q is compensated considering each phase separately and using energy storage elements. On the other hand, H is compensated with passive filters that need capacitors and inductors, also, energy storage components. With the new method, part of H (that depends on \tilde{q}) and \bar{q} are compensated for with active power filters without energy storage elements [2]. The other part of H (that depends on \tilde{p}) needs capacitors or inductors to be compensated [8].

The new method has the great advantage that changes in the harmonic or reactive power required by the load can be immediately compensated.

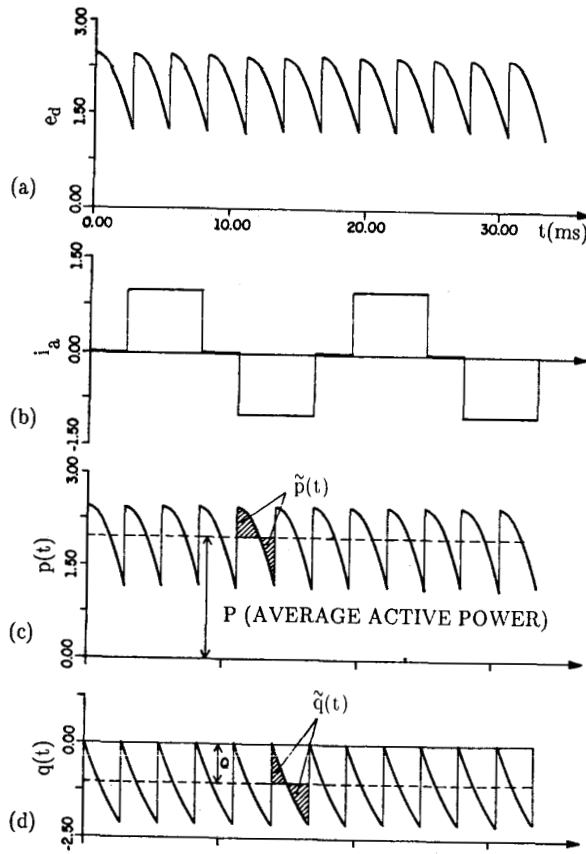


Fig. 3 - (a) three-phase rectifier output voltage e_d (firing angle $\alpha=30^\circ$); (b) rectifier a-phase input current i_a ; (c) instantaneous real power $p(t)$; (d) instantaneous imaginary power $q(t)$.

On the other hand, as the power semiconductor devices voltage and current ratings are increasing rapidly and with decreasing prices, the use of active power filters without energy storage elements for harmonic and reactive power compensation may become an interesting solution also from the economic point of view [11].

SIMULATED EXAMPLE

To understand even better the meaning of each power component the simulation results performed with a conventional three-phase controlled full-bridge rectifier is presented. Fig. 3 shows the rectifier output voltage e_d , the a-phase input current, the instantaneous real power $p(t)$ and the instantaneous imaginary power $q(t)$. This rectifier does not generate zero-sequence components. For this simulation it was considered that the rms value of the input voltage (see (34)) is equal to 1 p.u., the amplitude of the rippleless dc current is equal also to 1 p.u., the firing angle $\alpha = 30^\circ$ and overlap angle equal to zero. From this figure it is easy to understand [8] the meaning of $\tilde{p}(t)$ and $\tilde{q}(t)$.

Fig. 4(a) shows the power supply current after compensation of the total reactive current component ($q = \bar{q} + \tilde{q}$). Although some harmonic component is still present in the source current there is no more reactive components in it. The harmonic appearing in this figure is dependent on $\tilde{p}(t)$, the alternating (harmonic) real power. Fig. 4(b) shows a similar figure as in case (a), with the difference that in this case only the harmonic reactive current $i_{a\tilde{q}}$ has been

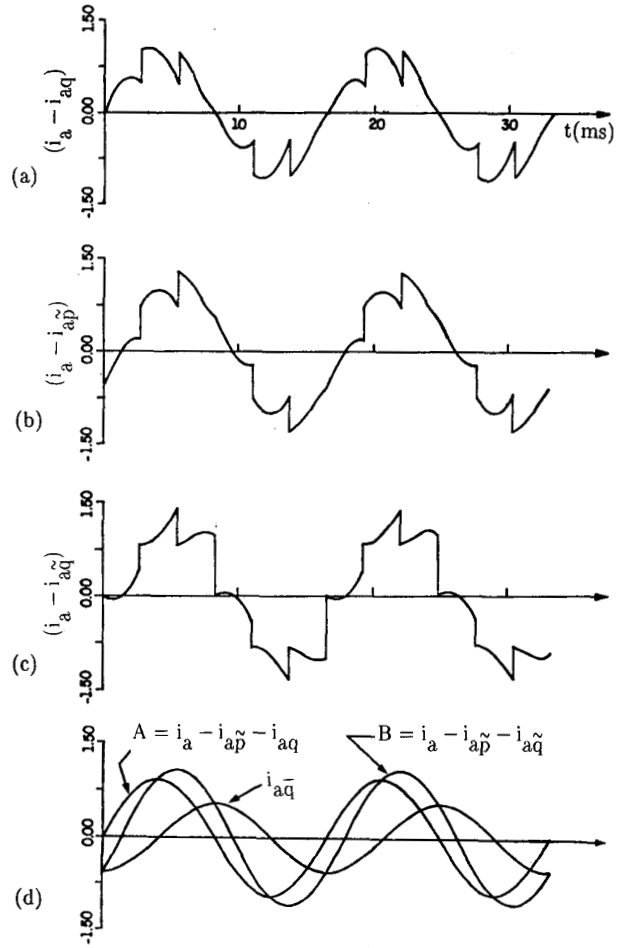


Fig. 4 - Currents: (a) $i_a - i_{aq}$; (b) $i_a - i_{a\tilde{q}}$; (c) $i_a - i_{a\tilde{q}}$; (d) $i_{a\tilde{q}}$; $i_a - i_{a\tilde{q}} - i_{aq}$ and $i_a - i_{a\tilde{q}} - i_{a\tilde{q}}$.

eliminated. Both compensations in Fig. 4(a) and (b) can be made without using energy storing devices.

Fig. 4(c) shows a different compensation. In this case only the alternating real power is eliminated. That is, with this compensation the power supply instantaneous real power becomes just a constant value, since $\tilde{p}(t)$ is eliminated. This kind of filtering may be important in applications where the power supply is a generator and can not have a torque ripple due to $\tilde{p}(t)$, because it may cause mechanical stress or mechanical vibration. Here we can say that $\tilde{p}(t)$ gives the value of the "active" alternating power that the rectifier is absorbing or generating. What is more interesting is that since $\tilde{p}(t)$ represents the effective transport of energy at high frequency it may be translated in mechanical vibration in the generator or other loads close to the rectifier, causing mechanical stress or vibration.

Fig. 4(d) shows the current $i_{a\tilde{q}}$ that corresponds to the conventional reactive current. The waveform B is for the case when harmonics due to $\tilde{p}(t)$ and $\tilde{q}(t)$ are eliminated and A shows the current waveform for the case when harmonic ($\tilde{p}(t) + \tilde{q}(t)$) and reactive (fundamental) power (\bar{q}) are compensated. It is interesting to confirm that by eliminating $\tilde{p}(t)$ and $\tilde{q}(t)$ the current becomes a sinusoidal wave.

Moreover, when \bar{q} , $\tilde{p}(t)$ and $\tilde{q}(t)$ are compensated (waveform A) the current becomes in phase with the α -phase voltage.

CONCLUSION

This paper, based on reference [1] and [2], presented a detailed analysis of the Instantaneous Power Theory in systems with non-linear loads and tried to explain clearly the meaning of each power in α - β -0 coordinates. Of course, it is also valid for 3-phase system. The authors believe that this theory is the base for designing active power filters with different functions as shown in the example presented. Also the authors believe that based on this theory new generation of electric system power measurement and monitoring instrumentation should be developed.

The authors are working on the extension of this theory for the case of unbalanced systems and plans to publish new compensation methods for three-phase 4-wires systems in the near future.

ACKNOWLEDGEMENT

The authors would like to express their gratitude to Prof. Hirofumi Akagi for his encouragement and helpful discussions. The authors would like to thank Mr. Márcio Magalhães for his help in performing the simulations presented here. The scholarship granted to M. Aredes by CEPTEL is acknowledged.

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BIOGRAPHIES

E.H. Watanabe (M'76) was born in Rio de Janeiro State, Brazil, on November 07, 1952. He received the B.Sc. in electronic engineering and M.Sc. in electric engineering in 1975 and 1976, respectively, from the Federal University of Rio de Janeiro. In 1981 he got the D.Eng. degree from Tokyo Institute of Technology, Japan.

Since 1981 he has been an Associate Professor at COPPE/Federal University of Rio de Janeiro, where he teaches Power Electronics. His main fields of interests are converters analysis, modeling and design, ac and dc drives and theory and application of active power filters.

Dr. Watanabe is a member of the IEE-Japan, The Brazilian Society for Automatic Control and is presently the Chairman of the IEEE-IAS Rio de Janeiro Chapter.

R.M. Stephan received the B.Sc. degree in electrical engineering from Instituto Militar de Engenharia (IME), Rio de Janeiro, in 1976, the M.Sc. degree from Universidade Federal do Rio de Janeiro (UFRJ/COPPE) in 1980, and the Dr.-Ing. degree from Ruhr-Universität Bochum, Germany, in 1985. During 1977 he worked as an engineer at Furnas Centrais Elétricas, Rio de Janeiro. Since 1978 he is at Universidade Federal do Rio de Janeiro (UFRJ). From 1982 to 1985 he was on leave from UFRJ as a DAAD (Deutscher Akademischer Austauschdienst) scholar at Ruhr-Universität Bochum. His research interests are in the fields of control of electrical drives, power electronics and power systems.

M. Aredes was born in São Paulo State, Brazil, on August 14, 1961. He received the B.Sc. degree in electrical engineering from Fluminense Federal University, Rio de Janeiro State, in 1984. From 1985 to 1988 he worked as a commissioning and project engineer at Itaipu HVDC Transmission System. From 1988 to 1991 he worked as management engineer in SCADA Project of Itaipu Power Plant. At present he is working on his master thesis at COPPE/Federal University of Rio de Janeiro, in the field of Active Power Filters.

NOMENCLATURE

ω	line frequency;
$p_{3\phi}$	instantaneous three-phase active power;
$p(t)$	instantaneous real power;
$q(t)$	instantaneous imaginary power;
$p_0(t)$	instantaneous zero-sequence power;
$p_i(t)$	instantaneous power at phase i , $i = a, b, c, \alpha, \beta$ or 0;
$p_{\alpha p}(t)$	instantaneous active power at phase α ;
$p_{\beta p}(t)$	instantaneous active power at phase β ;
$p_{\alpha q}(t)$	instantaneous reactive power at phase α ;
$p_{\beta q}(t)$	instantaneous reactive power at phase β ;
$\bar{p}(t)$	average value of $p(t)$;
$\bar{q}(t)$	average value of $q(t)$;
$\tilde{p}(t)$	alternating components of $p(t)$, $p(t) = \bar{p}(t) + \tilde{p}(t)$;
$\tilde{q}(t)$	alternating components of $q(t)$, $q(t) = \bar{q}(t) + \tilde{q}(t)$;
P	average active power;
Q	reactive power;
$P_{3\phi}$	3-phase average active power;
$Q_{3\phi}$	3-phase reactive power;
$v_i(t)$	instantaneous voltage at phase i , $i = a, b, c, \alpha, \beta$ or 0;
$i_i(t)$	instantaneous current at phase i , $i = a, b, c, \alpha, \beta$ or 0;
I_n	rms value of the n -th current harmonic;
\dot{V}_i	voltage phasor corresponding to phase i ;
\dot{I}_i	current phasor corresponding to phase i ;
V_n	rms value of n -th voltage harmonic;
Δ	$v_\alpha^2 + v_\beta^2$.